Paper Reference(s)

6674/01

Edexcel GCE

Further Pure Mathematics FP1
Advanced Level

Wednesday 20 June 2007 − Afternoon
Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6674), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N26111A

1. Find the set of values of x for which

$$\frac{x+1}{2x-3} < \frac{1}{x-3}.$$

(7)

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2. $$\frac{\mathrm{d}y}{\mathrm{d}x} - y \tan x = 2 \sec^3 x.$$

Given that $y = 3$ at $x = 0$, find y in terms of x.

(7)

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3. (*a*) Show that $(r + 1)^3 - (r - 1)^3 \equiv 6r^2 + 2$.

(2)

(*b*) Hence show that $\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n + 1)(2n + 1)$.

(5)

(*c*) Show that $\sum_{r=n}^{2n} r^2 = \frac{1}{6} n(n + 1)(an + b)$, where a and b are constants to be found.

(4)

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4. $$\mathrm{f}(x) = x^3 + 8x - 19.$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ has only one real root.

(3)

(*b*) Show that the real root of $\mathrm{f}(x) = 0$ lies between 1 and 2.

(2)

(*c*) Obtain an approximation to the real root of $\mathrm{f}(x) = 0$ by performing two applications of the Newton-Raphson procedure to $\mathrm{f}(x)$, using $x = 2$ as the first approximation. Give your answer to 3 decimal places.

(4)

(*d*) By considering the change of sign of $\mathrm{f}(x)$ over an appropriate interval, show that your answer to part (*c*) is accurate to 3 decimal places.

(2)

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5. For the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2x(x + 3),$$

find the solution for which at $x = 0$, $\frac{dy}{dx} = 1$ and $y = 1$.

(12)

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6. $$z = \sqrt{3} - i.$$

z^* is the complex conjugate of z.

(*a*) Show that $\frac{z}{z^*} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$.

(3)

(*b*) Find the value of $\left|\frac{z}{z^*}\right|$.

(2)

(*c*) Verify, for $z = \sqrt{3} - i$, that $\arg \frac{z}{z^*} = \arg z - \arg z^*$.

(4)

(*d*) Display z, z^* and $\frac{z}{z^*}$ on a single Argand diagram.

(2)

(*e*) Find a quadratic equation with roots z and z^* in the form $ax^2 + bx + c = 0$, where a, b and c are real constants to be found.

(2)

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7. (*a*) Sketch the curve C with polar equation

$$r = 5 + \sqrt{3}\cos\theta, \quad 0 \le \theta < 2\pi.$$

(2)

(*b*) Find the polar coordinates of the points where the tangents to C are parallel to the initial line $\theta = 0$. Give your answers to 3 significant figures where appropriate.

(6)

(*c*) Using integration, find the area enclosed by the curve C, giving your answer in terms of π.

(6)

TOTAL FOR PAPER: 75 MARKS

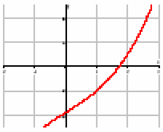
END

June 2007
6674 Further Pure Mathematics FP1
Mark Scheme

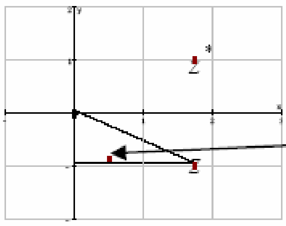
Question number	Scheme		Marks
1.	$1\frac{1}{2}$ and 3 are 'critical values', e.g. used in solution, or both seen as asymptotes		B1
	$(x+1)(x-3) = 2x-3 \;\Rightarrow\quad x(x-4) = 0$		
	$x = 4, \quad x = 0$	M1: attempt to find at least one other critical value	M1 A1, A1
	$0 < x < 1\frac{1}{2}, \quad 3 < x < 4$	M1: An inequality using $1\frac{1}{2}$ or 3	M1 A1, A1 (7)
			7
	First M mark can be implied by the two correct values, but otherwise a method must be seen. (The method may be graphical, but either ($x =$) 4 or ($x =$) 0 needs to be clearly written or used in this case). Ignore 'extra values' which might arise through 'squaring both sides' methods. $\leq$ appearing: maximum one A mark penalty (final mark).		

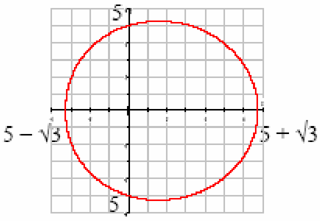
Question number	Scheme	Marks
2.	Integrating factor $e^{\int -\tan x\,dx} = e^{\ln(\cos x)}\ \left(\text{or } e^{-\ln(\sec x)}\right), \qquad = \cos x \left(\text{or } \frac{1}{\sec x}\right)$	M1, A1
	$\left(\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x\right)$	
	$y\cos x = \int 2\sec^2 x\,dx$ (or equiv.) $\left(\text{Or : } \frac{d}{dx}(y\cos x) = 2\sec^2 x\right)$	M1 A1(ft)
	$y\cos x = 2\tan x\ (+C)$ (or equiv.)	A1
	$y = 3$ at $x = 0$: $\qquad C = 3$	M1
	$y = \frac{2\tan x + 3}{\cos x}$ (Or equiv. in the form $y = f(x)$)	A1 (7)
		7
	1st M: Also scored for $e^{\int \tan x\,dx} = e^{-\ln(\cos x)}\ \left(\text{or } e^{\ln(\sec x)}\right)$, then A0 for $\sec x$. 2nd M: Attempt to use their integrating factor (requires one side of the equation 'correct' for their integrating factor). 2nd A: The follow-through is allowed <u>only</u> in the case where the integrating factor used is $\sec x$ or $-\sec x$. $\left(y\sec x = \int 2\sec^4 x\,dx\right)$ 3rd M: Using $y = 3$ at $x = 0$ to find a value for C (dependent on an integration attempt, however poor, on the RHS). <u>Alternative</u> 1st M: Multiply through the given equation by $\cos x$. 1st A: Achieving $\cos x\frac{dy}{dx} - y\sin x = 2\sec^2 x$. (Allowing the possibility of integrating by inspection).	

Question number	Scheme	Marks
3.	(a) $(r+1)^3 = r^3 + 3r^2 + 3r + 1$ and $(r-1)^3 = r^3 - 3r^2 + 3r - 1$	M1
	$(r+1)^3 - (r-1)^3 = 6r^2 + 2$ (*)	A1cso (2)
	(b) $r = 1$: $2^3 - 0^3 = 6(1^2) + 2$ $r = 2$: $3^3 - 1^3 = 6(2^2) + 2$: : : : : : : : : : : $r = n$: $(n+1)^3 - (n-1)^3 = 6n^2 + 2$ M: Differences: at least first, last and one other.	M1 A1
	Sum: $(n+1)^3 + n^3 - 1 = 6\sum r^2 + 2n$ M: Attempt to sum at least one side. $\left(6\sum r^2 = 2n^3 + 3n^2 + n\right)$	M1 A1
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ (Intermediate steps are not required) (*)	A1cso (5)
	(c) $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$, $= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2n-1)$	M1, A1
	$= \frac{1}{6}n\left(\left(16n^2 + 12n + 2\right) - \left(2n^2 - 3n + 1\right)\right)$	M1
	$= \frac{1}{6}n(n+1)(14n+1)$	A1 (4)
		11
	(b) 1st A: Requires first, last and one other term correct on both LHS and RHS (but condone 'omissions' if following work is convincing). (c) 1st M: Allow also for $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$. 2nd M: Taking out (at some stage) factor $\frac{1}{6}n$, and multiplying out brackets to reach an expression involving n^2 terms.	

Question number	Scheme	Marks	
4.	(a) $f'(x) = 3x^2 + 8$ $\quad 3x^2 + 8 = 0\ldots\ldots$ or $3x^2 + 8 > 0\ldots\ldots$	M1	
	Correct derivative and, e.g., 'no turning points' or 'increasing function'.	A1	
	 Simple sketch, (increasing, crossing positive x-axis) (or, if the M1 A1 has been scored, a reason such as 'crosses x-axis only once').	B1	(3)
	(b) Calculate $f(1)$ and $f(2)$ (Values must be seen)	M1	
	$f(1) = -10$, $f(2) = 5$, Sign change, $\therefore$ Root	A1	(2)
	(c) $x_1 = 2 - \dfrac{f(2)}{f'(2)}$, $\quad = 2 - \dfrac{5}{20}$ $\quad (= 1.75)$	M1, A1	
	$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$, $\quad \left(= 1.75 - \dfrac{0.359375}{17.1875}\right)$ $\quad = 1.729$ (ONLY) (α)	M1, A1	(4)
	(d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ (or a 'tighter' interval that gives a sign change).	M1	
	$f(1.7285) = -0.0077\ldots$ and $f(1.7295) = 0.0092\ldots$, $\therefore$ Accurate to 3 d.p.	A1	(2)
			11
	(a) M: Differentiate and consider sign of $f'(x)$, or equate $f'(x)$ to zero. Alternative: M1: Attempt to rearrange as $x^3 - 19 = -8x$ or $x^3 = 19 - 8x$ (condone sign slips), and to sketch a cubic graph and a straight line graph. A1: Correct graphs (shape correct and intercepts 'in the right place'). B1: Comment such as "one intersection, therefore one root"). (c) 1st A1 can be implied by an answer of 1.729, provided N.R. has been used. Answer only: No marks. The Newton-Raphson method must be seen. (d) For A1, correct values of f(1.7285) and f(1.7295) must be seen, together with a conclusion. If only 1 s.f. is given in the values, allow rounded (e.g. -0.008) or truncated (e.g. -0.007) values.		

Question number	Scheme	Marks
5.	C.F. $m^2+3m+2=0$ $\quad m=-1$ and $m=-2$	M1
	$y = Ae^{-x}+Be^{-2x}$	A1 (2)
	P.I. $y = cx^2+dx+e$	B1
	$\frac{dy}{dx}=2cx+d,\ \frac{d^2y}{dx^2}=2c$ $\quad 2c+3(2cx+d)+2(cx^2+dx+e)\equiv 2x^2+6x$	M1
	$2c=2$ $\quad c=1$ $\quad$ (One correct value)	A1
	$6c+2d=6$ $\quad d=0$	
	$2c+3d+2e=0$ $\quad e=-1$ $\quad$ (Other two correct values)	A1
	General soln: $y=Ae^{-x}+Be^{-2x}+x^2-1$ $\quad$ (Their C.F. + their P.I.)	A1ft (5)
	$x=0,\ y=1:\ \ 1=A+B-1$ $\quad (A+B=2)$	M1
	$\frac{dy}{dx}=-Ae^{-x}-2Be^{-2x}+2x,\quad x=0,\ \frac{dy}{dx}=1:$ $\quad 1=-A-2B$	M1
	Solving simultaneously: $\quad A=5$ and $B=-3$	M1 A1
	Solution: $\quad y=5e^{-x}-3e^{-2x}+x^2-1$	A1 (5)
		12
	1st M: Attempt to solve auxiliary equation. 2nd M: Substitute their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the D.E. to form an identity in x with unknown constants. . 3rd M: Using $y=1$ at $x=0$ in their general solution to find an equation in A and B. 4th M: Differentiating their general solution (condone 'slips', but the powers of each term must be correct) and using $\frac{dy}{dx}=1$ at $x=0$ to find an equation in A and B. 5th M: Solving simultaneous equations to find both a value of A and a value of B.	

Question number	Scheme	Marks
6.	(a) $z^* = \sqrt{3} + \mathrm{i}$	B1
	$\dfrac{z}{z^*} = \dfrac{(\sqrt{3}-\mathrm{i})(\sqrt{3}-\mathrm{i})}{(\sqrt{3}+\mathrm{i})(\sqrt{3}-\mathrm{i})} = \dfrac{3-2\sqrt{3}\mathrm{i}-1}{3+1}, \quad = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$ (*)	M1, A1cso (3)
	(b) $\left\lvert \dfrac{z}{z^*} \right\rvert = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\pm\sqrt{3}}{2}\right)^2}, \quad = 1$ $\quad \left[\text{Or: } \left\lvert \dfrac{z}{z^*} \right\rvert = \dfrac{\lvert z \rvert}{\lvert z^* \rvert} = \dfrac{\sqrt{3+1}}{\sqrt{3+1}}, \quad = 1 \right]$	M1, A1 (2)
	(c) $\arg(w) = \arctan\left(\pm \dfrac{\text{imag}(w)}{\text{real}(w)}\right)$ or $\arg(w) = \arctan\left(\pm \dfrac{\text{real}(w)}{\text{imag}(w)}\right)$, where w is z or z^* or $\dfrac{z}{z^*}$	M1
	$\arg\left(\dfrac{z}{z^*}\right) = \arctan\left(\dfrac{-\sqrt{3}/2}{1/2}\right) \quad = -\dfrac{\pi}{3}$	A1
	$\arctan\left(\dfrac{-1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$ and $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = \dfrac{\pi}{6}$ (Ignore interchanged z and z^*)	A1
	$\arg z - \arg z^* = -\dfrac{\pi}{6} - \dfrac{\pi}{6} = -\dfrac{\pi}{3} = \arg\left(\dfrac{z}{z^*}\right)$	A1 (4)
	(d) 	
	z and z^* (Correct quadrants, approx. symmetrical)	B1
	$\dfrac{z}{z^*}$ (Strictly inside the triangle shown here)	B1 (2)
	(e) $(x - (\sqrt{3} - \mathrm{i}))(x - (\sqrt{3} + \mathrm{i}))$	M1
	Or: Use sum of roots $\left(= \dfrac{-b}{a}\right)$ and product of roots $\left(= \dfrac{c}{a}\right)$.	
	$x^2 - 2\sqrt{3}x + 4$	A1 (2) **13**
	(a) M: Multiplying both numerator and denominator by $\sqrt{3} - \mathrm{i}$, and multiplying out brackets with <u>some</u> use of $\mathrm{i}^2 = -1$. (b) Answer 1 with no working scores both marks. (c) Allow work in degrees: $-60°$, $-30°$ and $30°$ Allow arg between 0 and 2π: $\dfrac{5\pi}{3}$, $\dfrac{11\pi}{6}$ and $\dfrac{\pi}{6}$ (or 300°, 330° and 30°). Decimals: Allow marks for awrt -1.05 (A1), -0.524 and 0.524 (A1), but then A0 for final mark. (Similarly for 5.24 (A1), 5.76 and 0.524 (A1)). (d) Condone wrong labelling (or lack of labelling), if the intention is clear.	

Question number	Scheme	Marks	
7.	(a) 		
	Shape (closed curve, approx. symmetrical about the initial line, in all 'quadrants' and 'centred' to the right of the pole/origin).	B1	
	Scale (at least one correct 'intercept' r value… shown on sketch or perhaps seen in a table). (Also allow awrt 3.27 or awrt 6.73).	B1	(2)
	(b) $y = r\sin\theta = 5\sin\theta + \sqrt{3}\sin\theta\cos\theta$	M1	
	$\frac{dy}{d\theta} = 5\cos\theta - \sqrt{3}\sin^2\theta + \sqrt{3}\cos^2\theta \quad \left(= 5\cos\theta + \sqrt{3}\cos 2\theta\right)$	A1	
	$5\cos\theta - \sqrt{3}\left(1-\cos^2\theta\right) + \sqrt{3}\cos^2\theta = 0$	M1	
	$2\sqrt{3}\cos^2\theta + 5\cos\theta - \sqrt{3} = 0$		
	$\left(2\sqrt{3}\cos\theta - 1\right)\left(\cos\theta + \sqrt{3}\right) = 0 \qquad \cos\theta = \ldots \quad (0.288\ldots)$	M1	
	$\theta = 1.28$ and 5.01 (awrt) (Allow ± 1.28 awrt) $\left(\text{Also allow } \pm\arccos\frac{1}{2\sqrt{3}}\right)$	A1	
	$r = 5 + \sqrt{3}\left(\frac{1}{2\sqrt{3}}\right) = \frac{11}{2}$ (Allow awrt 5.50)	A1	(6)
	(c) $r^2 = 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta$	B1	
	$\int 25 + 10\sqrt{3}\cos\theta + 3\cos^2\theta\, d\theta = \underline{\frac{53\theta}{2} + 10\sqrt{3}\sin\theta} + \underline{\underline{3\left(\frac{\sin 2\theta}{4}\right)}}$	M1 <u>A1ft</u> <u>A1ft</u>	
	(ft for integration of $(a + b\cos\theta)$ and $c\cos 2\theta$ respectively)		
	$\frac{1}{2}\left[25\theta + 10\sqrt{3}\sin\theta + \frac{3\sin 2\theta}{4} + \frac{3\theta}{2}\right]_0^{2\pi} = \ldots\ldots$	M1	
	$= \frac{1}{2}(50\pi + 3\pi) = \frac{53\pi}{2}$ or equiv. in terms of π.	A1	(6)
			14
	(b) 2nd M: Forming a quadratic in $\cos\theta$. 3rd M: Solving a 3 term quadratic to find a value of $\cos\theta$ (even if called θ). <u>Special case</u>: Working with $r\cos\theta$ instead of $r\sin\theta$: 1st M1 for $r\cos\theta = 5\cos\theta + \sqrt{3}\cos^2\theta$ 1st A1 for derivative $-5\sin\theta - 2\sqrt{3}\sin\theta\cos\theta$, then no further marks. (c) 1st M: Attempt to integrate at least one term. 2nd M: Requires use of the $\frac{1}{2}$, correct limits (which could be 0 to 2π, or $-\pi$ to π, or 'double' 0 to π), and subtraction (which could be implied).		